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Can the constructive empiricist be a nominalist? Quasi-truth, commitment and consistency

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Abstract

In this paper, I explore Rosen's (1994) 'transcendental' objection to constructive empiricism—the argument that in order to be a constructive empiricist, one must be ontologically committed to just the sort of abstract, mathematical objects constructive empiricism seems committed to denying. In particular, I assess Bueno's (1999) 'partial structures' response to Rosen, and argue that such a strategy cannot succeed, on the grounds that it cannot provide an adequate metalogic for our scientific discourse. I conclude by arguing that this result provides some interesting consequences in general for anti-realist programmes in the philosophy of science.

Keywords: Constructive empiricism; Mathematical fictionalism; Partial structures; Modality

1. Introduction: abstract commitments

There is, *prima facie*, one important respect in which the constructive empiricist seems committed to believing in the existence of abstract objects. Constructive empiricism, recall, is the view that 'science aims to give us theories that are empirically adequate; and acceptance of a theory involves as belief only that it is empirically adequate' (van Fraassen, 1980, p. 12). But as Rosen (1994, p. 165) argues, for the constructive empiricist to believe that a theory is empirically adequate is for him to believe that the theory exists, and that it possesses the complex relational property of empirical adequacy. But in which case,

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abstract ontological commitment immediately follows, since according to van Fraassen, a scientific theory is an abstract mathematical object.[1]

Unfortunately, there also seems to be an important respect in which belief in the existence of abstract mathematical objects is incompatible with constructive empiricism. As Rosen (1994, p. 164) puts it, constructive empiricism appears to entail a form of *nominalism*: abstract objects are unobservable objects, and experience—the source of all the constructive empiricist's knowledge—cannot tell us whether abstract objects exist or not. Indeed, van Fraassen (1974) himself makes just this point: the two lands of Oz and Id disagree on the existence of sets and abstract objects, but both live happily ever after *precisely because* the existence of abstract objects makes no empirical difference. It follows then, according to Rosen, that 'just as [the constructive empiricist] suspends judgement on what his theory says about unobservable physical objects, he should suspend judgement on what they say about the abstract domain' (1994, p. 164). Indeed, here one could surely generalise an argument due to Ladyman (2000): commenting on the possible combination of constructive empiricism and modal realism, Ladyman remarks that it would be 'bizarre to suggest that we do not know about electrons merely because they are unobservable, but that we do know about non-actual possibilia' (p. 855). Arguably, it would be equally 'bizarre' for the constructive empiricist to claim that he knows about, and can believe in, abstract mathematical objects, yet continue to maintain his agnosticism about electrons.

It seems then that the constructive empiricist faces the following dilemma. On the one hand, in order to even state his position, he appears to be committed to believing in the existence of abstract objects. Yet on the other hand, in order to remain consistent with the epistemological scepticism that drives constructive empiricism, he appears to be committed to not believing in the existence of abstract objects: the *most* he can manage is a characteristically agnostic attitude about the matter.

Monton & van Fraassen (2003, p. 412) discuss Rosen's objection, somewhat tangentally, in a footnote. They concede that there is a potential tension between van Fraassen's mathematical nominalism, and his use of mathematics in articulating constructive empiricism, but argue that the objection holds on 'the supposition that mathematics is intelligible only if we can view it as a true story about certain kinds of things', which is not a supposition shared by all philosophies of mathematics. This is certainly true; but as a response to Rosen, it amounts to little more than the claim that one cannot solve the problems of the philosophy of mathematics *en passant* in the philosophy of science. In contrast to Monton and van Fraassen, I believe that Rosen's objection is both sufficiently dangerous for, and potentially illuminating of, constructive empiricism to require more than a promissory note in response. Indeed, the central thought underlying this paper is not only that one cannot make significant progress in the philosophy of science without due attention to the philosophy of mathematics, but that careful attention to the philosophy of science can also offer significant progress in the philosophy of mathematics.

The focus of this paper however will be on one specific response to Rosen's problem: Bueno's (1999) 'partial structures' approach. Bueno's proposal, roughly speaking, is for the constructive empiricist to adopt a *fictionalist* attitude towards abstract mathematical

[1] More specifically, van Fraassen takes a theory to be a set of Suppesian models, which are themselves some kind of abstract, set theoretic entity; see van Fraassen (1980), pp. 64–69; (1989), pp. 217–232.

objects, suitably formulated for empiricist purposes (hence the partial structures). The idea is that the constructive empiricist can then argue that his commitment to abstract objects need not entail belief in their existence: in fact, his commitment to abstract objects is actually consistent with their *non-existence*, since the usefulness of the various claims made about abstract objects can be accounted for in some way other than their truth.

Such a response clearly relieves the constructive empiricist from the burden of believing in the existence of abstract objects; and moreover, it seems in line with van Fraassen's own thoughts on the matter: he notes that he has 'not worked out a nominalist philosophy of mathematics... yet [it is] clear that it would have to be a fictionalist account' van Fraassen, (1985, p. 303). But it remains an open question as to how successful the combination of constructive empiricism and mathematical fictionalism really is. Having outlined Bueno's position, I shall note two initial reservations I have about his overall strategy: an issue concerning the motivation for the position, and an objection from bad company. My main concern however will be over the notion of *consistency*. Quite simply, I do not believe that fictionalism has the resources to provide an adequate account of what it means for a theory to be consistent. Such a result would be problematic for any defence of constructive empiricism, but in the case of Bueno's strategy it is lethal: for as we shall see, the fictionalist programme depends essentially upon an adequate account of consistency.

In the final part of this paper then, I shall discuss the issue of abstract objects, ontological commitment and the notion of consistency more generally. I shall argue however that, contrary to Rosen's argument, constructive empiricism does offer an attractive, anti-realist account of these issues, and does so in a way that promises a substantial advantage over other anti-realist positions.

2. Empiricism, fictionalism and quasi-truth

Bueno's proposal is to 'reformulate in empiricist terms, the fictionalist account put forward by Hartry Field' (1999, p. S475). Field's (1980, 1989) programme, as is well known, is an attempt to navigate the two horns of Benacerraf's (1973) dilemma. On the one hand, it attempts to provide a satisfactory account of mathematical truth by adopting a realist semantics for mathematical discourse; while on the other, it attempts to avoid the epistemological problems of how we could know whether a mathematical statement was true by denying the existence of mathematical objects, and thus concluding that in fact all mathematical statements are *false* (or trivially true in the case of universal quantification).

Field's strategy thus avoids Benacerraf's dilemma, but it brings to the fore the problem of the *applicability* of mathematics. Crucially, the fictionalist needs to account for the successful utilisation of mathematics—in particular within the natural sciences—given that mathematical statements are supposed to be false. In response, Field (1982, p. 51) argues that the usefulness of mathematical theories has nothing to do with their truth. Instead, it depends upon whether they are *conservative*, which is a strong form of consistency. Basically, a mathematical theory will be conservative for a particular subject matter iff it is consistent with every internally consistent, non-mathematical theory about that subject matter. More formally, for any theory T that is not itself explicitly mathematical, let T^* be the result of restricting all variables of T so that they are non-mathematical. Then, a mathematical theory M is conservative iff for any such T, if T is consistent, then so is $(T^* + M)$. A conservative theory adds nothing new: $(T^* + M)$ will not produce any non-mathematical conclusions that do not follow from T alone. However, a conservative theory may make the facilitation of certain inferences *easier*, and this, according to Field, is where the usefulness of mathematics lies.

It seems then that a fictionalist strategy can provide a resolution to the difficulties encountered above. The constructive empiricist, recall, faced the dilemma of being committed to various abstract, mathematical objects, while at the same time being unable to believe in their existence. By adopting some form of fictionalism however, it seems that the constructive empiricist can argue that his commitment to various theories need not entail belief in the existence of abstract objects, since he can accommodate the usefulness of these theories merely by showing that their claims are conservative rather than true.

Before we begin to assess the prospects of the combination of constructive empiricism and fictionalism however, it is important to recognise two key moves made by Field. The first is the emphasis placed on the notion of conservativeness: since mathematical statements are strictly speaking *false* according to Field, he needs to show that mathematical statements have some property other than truth that can account for the great range of successful applications made of them. The conservativeness of mathematical theories will count for naught however, unless Field's second move also goes through. As the definition above makes clear, conservativeness can only be employed in conjunction with *non-mathematical* theories: a mathematical theory is conservative over a domain iff it is consistent with every internally consistent, non-mathematical theory of that domain. In other words, in order to show that mathematics is conservative for science, Field first needs to show that the theories of science can be successfully *nominalised*; that is, he needs to show that mathematics is *dispensable* to science, and that the theories of science in which we are interested can be formulated without any reference to mathematical objects.

But as Bueno (1999, p. S479) points out, both of these moves present an initial difficulty for the constructive empiricist. Firstly, part of Field's strategy for the nominalization of science—in particular his strategy for the nominalization of Newtonian mechanics—involves quantification over spacetime points. That is to say, part of Field's programme involves assuming a substantivalist view of spacetime, according to which regions of spacetime *exist*, independently to whether they are occupied or not (1980, pp. 34–36; 1989, pp. 171–180). The problem is of course that the constructive empiricist cannot endorse such ontological commitments: the constructive empiricist can, at most, be *agnostic* about the existence of spacetime. Or as Bueno puts it:

> in order for Field to be an *antirealist* in the philosophy of mathematics, he has to adopt a *realist* attitude in the philosophy of science. And of course, despite Field's arguments for substantivalism, unoccupied spacetime regions are precisely the sort of entities an empiricist cannot assume. (1999, p. S479)

The first task for Bueno's constructive empiricist fictionalism therefore is to reconstrue Field's nominalization strategy along more ontologically parsimonious lines.

The second difficulty with the combination of constructive empiricism and fictionalism—and the focus of this paper—concerns the notion of conservativeness. As Field points out, conservativeness is clearly a stronger notion than consistency: although we can deduce the *consistency* of a theory from its *truth*, a *true* theory is not necessarily a *conservative* theory.[2] Further, since we also cannot deduce the *truth* of a theory from its

[2] Although, of course, a *necessarily true* theory will be conservative.

conservativeness, it is important to note that conservativeness is also not weaker than truth (Field, 1980, pp. 16–19; 1989, p. 59). It follows then, notes Bueno, that 'Field is not countenancing a weaker aim of mathematics [than truth], but only a *different* one' (1999, p. S476). But again this raises difficulties for the constructive empiricist, since according to Bueno 'an important feature of the constructive empiricist's axiology is that the aim of science should be *weaker than truth*, otherwise the grounds for adopting an antirealist proposal are lost' (ibid., p. S478).

Similarly, the motivation for the fictionalist's countenance of conservativeness is also too strong for the constructive empiricist. The fictionalist, recall, argues that abstract, mathematical objects do not exist. The constructive empiricist, by contrast, is merely *agnostic* about their existence. For both these reasons then, the second task for Bueno's constructive empiricist fictionalism will be to countenance a more diluted notion of conservativeness in order to accommodate the constructive empiricist's weaker epistemic stance.

It is in response to both of these difficulties that Bueno appeals to the notions of *partial structure* and *quasi-truth*, as formulated by da Costa and French (da Costa, 1986; da Costa & French, 1989, 1990). Basically, a partial structure provides a rigorous way of accommodating incomplete knowledge of a domain within the standard set theoretic machinery. In particular, a partial structure consists of various partial relations, and a partial relation consists of three sets of ordered n-tuples: the set of n-tuples for which the relation holds, the set of n-tuples for which the relation does not hold, and the set of n-tuples *for which we do not know whether the relation holds or not*.

Tarskian truth is of course only defined for full structures; thus in order to extend the notion to cover partial structures, the intermediary notion of quasi-truth is introduced. A partial structure can be extended into a full structure by making all of its partial relations determinate; that is, for each n-tuple for which we do not know whether a particular relation holds, we simply stipulate whether it does or not. A sentence α can then be said to be quasi-true in a partial structure A iff there is a full structure B, which is a consistent extension of A, and α is true in B.

So how then does the introduction of quasi-truth help the combination of constructive empiricism and fictionalism? The problem was the notion of conservativeness, which as defined by Field was too strong for the constructive empiricist. The crucial point to note then is that quasi-truth is much weaker than truth: a quasi-true sentence α does not completely describe the domain in question, but merely the aspect of the domain modelled by the relevant partial structure. Thus, since there are numerous different ways a partial structure can be extended into a full structure, and since in some of these structures α may be false, it follows that quasi-truth is weaker than truth: true sentences will be quasi-true, but quasi-true sentences will not necessarily be true.

Subsequently, the notion of quasi-truth can be used to allow a weaker account of conservativeness. A mathematical theory M is said to be *weakly conservative* if it is quasi-true in a partial structure A with respect to a consistent body N of nominalistic claims. From this it follows that M is weakly conservative iff M is consistent with *some* internally consistent body of claims about the physical world (Bueno, 1999, p. S482). The notion of weak conservativeness is then much like Field's notion, with the important exception that weak conservativeness only requires consistency with *some* body of physical claims about the domain in question, whereas Field's notion requires consistency with *all* bodies of physical claims about the domain.

3. Assessing Bueno's strategy

I shall not discuss the technical details of Bueno's strategy here; suffice it to say I find his proposal to combine constructive empiricism and fictionalism ingenious, and his attempt to reformulate Field's programme in empiricist terms ultimately convincing.[3] However, even granting the success of Bueno's arguments, I wish here to raise a couple of general reservations about the combination of constructive empiricism and fictionalism.

The first concerns motivation. One of the difficulties Bueno highlights for combining constructive empiricism and fictionalism is that while the fictionalist denies the existence of mathematical objects, the constructive empiricist is committed to remaining merely agnostic about such matters; part of the purpose of the partial structures approach was therefore to accommodate this weaker epistemic stance within the fictionalist framework. But without additional argument, it is far from clear that the constructive empiricist *is* committed to agnosticism about the existence of mathematical objects. Constructive empiricism, recall, is the view that 'science aims to give us theories that are empirically adequate; and acceptance of a theory involves as belief only that it is empirically adequate' (van Fraassen, 1980, p. 12). As it stands, this is purely a thesis about the epistemological limits of science; subsequently, the constructive empiricist is only committed to agnosticism about mathematical objects if it turns out that our knowledge of mathematical objects falls within the scope of our scientific theories.[4]

This may seem a terminological point, but it does serve to somewhat undermine the motivation for Bueno's position. For on the one hand, fictionalism maintains that mathematics is *dispensable* to science: our scientific theories can be nominalised, and the mathematics relegated to nothing more than the status of a formalised shorthand. But if mathematics is dispensable to science, then clearly our knowledge of mathematics is independent of our scientific knowledge (indeed, Field argues that most of our mathematical knowledge is essentially a kind of *logical* knowledge). But then, if mathematical knowledge is independent of our scientific knowledge, there is no reason at all why the constructive empiricist is committed to extending his epistemic policy to the mathematical realm. Subsequently, if mathematics is dispensable to science, it is consistent for the constructive empiricist to take any attitude he likes towards the existence of mathematical objects, full-blown platonism notwithstanding. On the other hand of course, if mathematics is not dispensable to science, then we cannot rule out the possibility that science furnishes us with our mathematical knowledge. And if mathematical knowledge does lie within the bounds of science, then there is ample motivation for Bueno's approach. Unfortunately though, if mathematics is not dispensable to science, then fictionalism collapses, and Bueno's approach cannot even get off the ground.

The point then is simply this: in order for fictionalism to be tenable, mathematics must be dispensable to our scientific theories. But if mathematics is dispensable, then knowledge of mathematical objects no longer constitutes a pressing problem for the constructive

[3] For a full discussion of the prospects of reformulating constructive empiricism in terms of partial structures, see Bueno (1997).

[4] This is something of a simplification: more accurately, constructive empiricism is a view about the *aim* of science, which in itself is compatible with a range of epistemological positions towards scientific theories. Nevertheless, the point still stands: a view about the aim of science entails nothing about our attitudes towards mathematical objects unless such objects can be shown to be within the scope of our scientific theories.

empiricist. Consequently, Rosen's original objection is defused, since the constructive empiricist is no longer *committed* to the sort of anti-realist stance Bueno proposes: it is simply an additional component he may or may not choose to adopt.

Of course, even if it is entirely open for the constructive empiricist to adopt any attitude he likes towards mathematical objects, he must still say *something* about them. And given the broadly nominalist flavour of his position, it seems reasonable to suppose that the most attractive strategy will be the one with the least ontological commitments. Thus even if the tenability of fictionalism undermines its principle motivation—by showing that the problem it is meant to respond to isn't a problem after all—the combination of constructive empiricism and fictionalism may still seem an attractive proposal overall, on the grounds of ontological parsimony if nothing else.

However, it remains far from clear whether fictionalism *is* a tenable proposal. Since its initial exposition, Field's programme has been subject to a barrage of criticisms. It has been objected, for example, that mathematics cannot be shown to be conservative due to the incompleteness of arithmetic (Shapiro, 1983); and that since Field's nominalisation strategy presupposes an infinite number of spacetime points and other nominalist surrogates, it is just as ontologically extravagant as the platonistic formulations it is supposed to replace (Melia, 1998). I don't intend to offer a complete evaluation of Field's programme here; needless to say, the existence of such difficulties seriously limits the attractiveness of fictionalism for the constructive empiricist.[5]

One objection in particular is worth discussing in a little more detail. Malament (1982) has rejected the possibility of a complete nominalisation of science, on the grounds that quantum mechanics is irreducibly statistical, and that one cannot find adequate nominalist surrogates for the probabilities involved. The most plausible response to this problem is due to Balaguer (1998), who has suggested that a nominalisation strategy could be carried out in terms of physically real propensities. Interestingly however, this is a response Bueno explicitly rejects. Bueno (2003) argues that propensities are not nominalistically acceptable entities; that Balaguer's strategy constitutes an *interpretation*, rather than a nominalisation, of quantum mechanics; and perhaps most importantly, that Balaguer's account is fundamentally inconsistent with van Fraassen's (1991) modal frequency interpretation of quantum mechanics. In fact, Bueno goes as far as to conclude 'that the nominalisation of QM remains a major problem for the nominalist' (2003, p. 1435). Regardless then of what one makes of the other objections raised against Field, it seems that Bueno himself believes that fictionalism still requires a lot more work. Consequently, the current prospects for a combination of constructive empiricism and fictionalism appear to *fail by Bueno's own standards*. The position is not thereby refuted; but pending these (substantial) revisions, there appears to be little to recommend it.

4. Primitive modal consistency

The central argument I want to present against the combination of constructive empiricism and fictionalism however concerns the notion of conservativeness. Conservativeness (both Field's original notion, and Bueno's weakened notion) is defined in terms

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[5] In this paper, my discussion of mathematical fictionalism will be limited to Field's articulation of the position, since this is both the most fully developed account within the literature, and the one explicitly endorsed by Bueno. In what follows, I leave it open as to whether my criticisms apply to *all* forms of mathematical fictionalism.

of consistency; and following Tarski (1936), consistency is usually defined in terms of truth in a model:

> Γ is logically consistent iff there is at least one model in which all the members of Γ are true.

However, a model is a mathematical object, and thus according to the fictionalist, does not exist. This then makes the above definition useless: if there are no models, there will never be a model in which all the members of Γ are true; and thus nothing would ever be logically consistent. In the case of the constructive empiricist fictionalist of course, it would be more accurate to say that we cannot *believe* that such mathematical objects exist. The same problem still arises however, since if we do not believe that the right sort of model exists, then we cannot believe that the theory is consistent, and hence cannot defend the applicability of a mathematical theory in terms of its weak conservativeness. There is a problem then as to what the fictionalist *means* by a theory being consistent, if not that it has a model, and hence what it means for a theory to be (weakly) conservative.[6]

Field's (1984a) response is to take the notion of consistency to be neither syntactic nor semantic, but rather as a *primitively modal* notion; and this seems to be the best strategy for Bueno as well. The basic idea is, following Kreisel (1967), that we can begin with an intuitive notion of implication, taken as a primitive in a similar way to negation, conjunction and universal quantification. That is, the meaning of implication is not to be conveyed by a definition, but rather by specifying procedural rules involved in inferring with it. These will include positive rules for recognising implication, and sufficient for showing that if Γ derives B in a typical formalisation of first-order logic, then Γ implies B; and negative rules for recognising failures of implication, sufficient for showing that if Γ does not Tarski-imply B, then Γ does not imply B.

Once this is established, Field (1989, pp. 34–35) goes on to use the primitive notion of implication to define a 1-place operator $\Box_L$ for 'it is logically true that', such that:

$$\Box_L(A) =_{df.} (A \vee \neg A) \rightarrow A$$

and a 1-place operator $\Diamond_L$ for 'it is logically possible that', such that:

$$\Diamond_L(A) =_{df.} \neg\Box_L\neg(A)$$

Intuitively, $\Box_L$ should also obey the following laws:

$$\Box_L(A) \supset A$$
$$\Box_L(A \supset B) \supset (\Box_L(A) \supset \Box_L(B))$$

Also, since $\Box_L(A)$ is a logical axiom whenever A is a logical axiom:

$$\Box_L(A) \rightarrow \Box_L(\Box_L(A))$$

Clearly then, $\Box_L$ operates in the same way as the standard necessity operator $\Box$ in S4; thus by taking implication as a primitive rather than a syntactic or semantic notion, we are led

[6] Similar problems face a syntactic notion of consistency. To say that a theory is *syntactically* consistent is to assert the non-existence of certain formal derivations, that is, derivations that start with the axioms of the theory and end in contradiction. Yet we cannot produce *concrete* tokens for every possible derivation; subsequently, in order to avoid certain theories coming out as consistent by default, we must also invoke various non-actual derivations, that is, more abstract, mathematical objects. It follows then that since the fictionalist must also deny their existence, a syntactic notion of consistency is similarly useless for our present purposes.

to the idea that implication, and related notions such as logical truth, are modal notions of some kind.

Subsequently, instead of making the metalogical claim that a mathematical theory T is consistent, we can instead make the object-level claim that the conjunction of the axioms of T is *logically possible*, that is:

$$\Diamond_L AX_T$$

where AX_T is the conjunction of axioms of a finitely axiomatisable theory T. This then allows the fictionalist to find an object-level assertion instead of the metalogical assertion of weak conservativeness. Recall that a mathematical theory M is weakly conservative iff for some non-mathematical theory T, if T is consistent, so is $T^* + M$, where T^* is the result of restricting all the quantifiers of T to non-mathematical objects. The modal surrogate for this claim would then be of the form:

$$\Diamond_L AX_T \supset \Diamond_L((AX_T)^* \wedge AX_M)$$

Generalising for all such theories, including those that cannot be finitely axiomatised, Field introduces a universal substitutional quantifier, Π, which takes formulae as its substitution class. Following Gottlieb (1980; see also Field, 1984b), Field uses this device to represent an infinite conjunction of object-level claims about theories. Thus generalising, a mathematical theory M is weakly conservative iff:

$$\neg\Pi B\neg(\Diamond_L B \supset \Diamond_L(B^* \wedge AX_M))$$

5. Modal languages and expressive completeness

The fictionalist's utilisation of a primitive modal operator in discussions of logical consistency invites an immediate comparison with similar strategies in the treatment of modality. In much the same way that Field rejects the need for a model theoretic semantics, various philosophers, for example Fine (1977), have argued for the rejection of a possible worlds semantics. Subsequently, our modal notions are given by primitive operators in much the same way that Field argues our logical notions are given by primitive operators; and talk of and commitment to possible worlds is to be rejected in much the same way as Field rejects talk of and commitment to mathematical objects.

However, there is a serious problem of expressive adequacy for modal languages that do not quantify over possible worlds. The argument is that there are intuitively true modal statements that cannot be expressed without some kind of possible worlds semantics. Consequently, since we appear to be committed to the truth of these statements, and since these statements can only be expressed by quantifying over possible worlds, we also appear to be committed to the *existence* of possible worlds, *contra* the primitive modalist.

Consider for example the modal statement:

1. There could have been more things than there actually are.

This is clearly an example of an intuitively true modal statement. And indeed, it is easily expressed in a two-sorted extensional language, that is, a language that quantifies over both objects and possible worlds, as:

$$\exists w[\forall x(Exw^* \rightarrow Exw) \wedge \exists y(Eyw \wedge \neg Eyw^*)]$$

where E is a two-place predicate for 'exists at', which takes constants/variables of sort two and sort one in its first and second places respectively, and w^* represents the actual world.

However, as Hazen (1976) originally showed, statements such as (1) cannot be expressed in a language that uses only the primitive operators $\Box$ and $\Diamond$.[7] For on the one hand, in order to assert the existence of an individual at a world other than the actual world, it is necessary to use a quantifier inside the scope of a modal operator. Yet on the other hand, in order to say that this individual is distinct from any individual existing in the actual world, it is necessary to use a second quantifier, which would be inside the scope of the first modal operator, but not governed by it. For example, we might try to formulate (1) as:

$$\Diamond[\forall x(AEx \rightarrow Ex) \wedge \exists y \neg AEy]$$

where A is an *actuality* operator, such that $A\varphi$ is true in a model iff φ is true at the actual world in the model. But in this case, the universal quantifier only ranges over the individuals that appear in the possible world introduced by the possibility operator. Consequently, it no longer tells us whether a world contains everything the actual world contains, plus something more; it merely tells us that there is a world that contains everything it contains, plus something the actual world does not. And this is clearly not the content of 'there could have been more things that there are': it is a much weaker claim, since it can be satisfied by worlds that contain a tiny subset of the actual world, plus one non-actual individual.

We could try to solve this problem by 'unrestricting' the universal quantifier:

$$\Diamond\{[\Box\forall x(AEx \rightarrow Ex)] \wedge \exists y \neg AEy\}$$

This now reads as 'necessarily, for any x...', thus referring to all possible individuals, not just the ones under the scope of the possibility operator. Unfortunately, the subclause '$\forall x(AEx \rightarrow Ex)$' now falls under the scope of the necessity operator rather than the possibility operator. Consequently, this expression only asserts the necessity of the proposition 'everything is such that, if it exists at the actual world, then it exists'. But this is a trivial assertion, and one that no longer has anything to do with the original possibility operator; thus the attempt to compare two different domains fails.

Thus (1) is an example of an intuitively true modal assertion that it appears cannot be expressed without a possible worlds semantics. So it seems that any satisfactory modal language must be one committed to the existence of possible worlds in some sense. Crucially, an entirely analogous argument can also be ran against Field's programme, and by extension, against Bueno as well. For it seems that there are intuitively assertions we can make about *logical possibility*, yet which cannot be expressed in a language that does not quantify over *models*, for example:

2. It is *logically consistent* that there be more things than there actually are.

Clearly, (2) is an example of an intuitively true statement about logical consistency: after all, it is logically consistent that there should be all the things that there are, and there is

[7] Actually, (1) is due to Melia (1992), p. 38. Although Hazen presents several examples of modal statements that cannot be expressed in a primitive modal language, I shall for the sake of clarity focus on Melia's example. This, I hope, shall help make explicit exactly what difficulties face a primitive modalist such as Field. What follows however will of course apply *mutatis mutandis* for all the so-called 'Hazen-sentences'.

nothing inconsistent about adding one more thing. It should also be clear that such a statement could be easily expressed in a language that allows quantification over models, in much the same way that (1) can be expressed in a language that quantifies over worlds, for example, for any given mathematical model, there is a model of greater cardinality. And it should also be clear that for the same reasons that (1) cannot be expressed by a primitive modal language, (2) cannot be expressed by Field's primitive operators. The issue of expressive adequacy therefore possesses a serious challenge for the (constructive empiricist) fictionalist. If there are statements such as (2) that we are committed to holding true, and that are inexpressible without quantification over models, then it seems that the fictionalist is wrong to claim that he can do without commitment to mathematical objects. Consequently, it appears that neither Field nor Bueno has a primitive modal notion of consistency available, and thus cannot define conservativeness without invoking the very mathematical objects their projects seeks to reject.

6. An expanded modal language

One possible solution to the problem of expressive adequacy is to expand our primitive modal language. In the case of modality, the most extensive work in this field has been undertaken by Forbes (1989). Following Peacocke (1978), the idea is to introduce a denumerable number of subscripted operators of the form $\Diamond_n$, $\Box_n$, and A_n. These are best understood by examining their model theoretic clauses. We first extend the model for the language to include an ordered sequence of worlds *w*, where *w*[v/i] is the result of substituting v for the ith world in *w*, and w_n is the nth member of *w*. Then, when evaluating a formula governed by $\Box_n$ at some world w, the operator is understood as sending us to some other world w′ to evaluate the sub-formula governed by the operator, and as storing w′ in the nth place of *w*. If the sub-formula is governed by A_n, the sub-formula governed by A_n is evaluated from the world w′.

This now gives Forbes the resources he needs to express (1), in the form:

$$\Diamond_1\{[\Box(\forall x)(AEx \rightarrow A_1Ex)] \wedge (\exists y)\neg AEy\}$$

which is interpreted as follows. As before, we have 'unrestricted' the universal quantifier by prefixing it with a necessity operator. However, when we come to evaluate '$(AEx \rightarrow A_1Ex)$' the new, subscripted actuality operator tells us that the second 'Ex' is to be evaluated from the world referred to by the subscripted possibility operator. Thus we are no longer forced by the scope of the necessity operator to assert the trivial claim that every world contains what it contains; instead the subclause now says that if some possible individual exists at the actual world, then it exists at w, that is, w includes all the individuals that exist at the actual world. Finally, since '$(\exists y)\neg AEy$' is also under the scope of $\Diamond_1$, it says there is an individual at w that does not exist at the actual world, that is, w contains all the entities that exist at the actual world, and at least one more thing.

It seems then that the constructive empiricist fictionalist can also avoid the objection from expressive adequacy by similarly expanding his primitive modal language to include subscripted operators of the form $\Diamond_{Ln}$, $\Box_{Ln}$ and A_{Ln}. However, Melia (1992, 2003) has raised a number of objections to Forbes's strategy, arguing that in reality such an account actually *invokes* the possible worlds it seeks to do without. He argues that in a case of genuine ontological reduction by paraphrase, the purported paraphrase displays a significant difference in syntax to the original formulation. For example, the sentence:

The number of dogs is three

can be paraphrased so as to eliminate any reference to numbers:

$$\exists x \exists y \exists z[(Fx \wedge Fy \wedge Fz \wedge x \neq y \wedge x \neq z \wedge y \neq z \wedge \forall w(Fw \rightarrow w = x \vee w = y \vee w = z)]$$

where F is the property of being a dog. Such a paraphrase clearly displays different syntactic characteristics from the original sentence. Yet as Melia notes, Forbes's modal language actually displays remarkable structural similarities to the kind of two-sorted extensional languages he opposes. Consider again the quantified formulation and the primitive operator formulation: the only real noticeable difference between the two is that the second formulation uses an actuality operator, **A**Ex, instead of a possible world variable, $Ex\mathbf{w}^*$; and a subscripted possibility operator, $\Diamond_1$, instead of a possible world quantifier, $\exists w$:

$$\exists w[\forall x(Ex\mathbf{w}^* \rightarrow Exw) \wedge \exists y(Eyw \wedge \neg Ey\mathbf{w}^*)]$$

$$\Diamond_1\{[\Box(\forall x)(\mathbf{A}Ex \rightarrow A_1 Ex)] \wedge (\exists y)\neg \mathbf{A}Ey\}$$

This is made explicit when we come to construct a translation schema between the two. If we write Forbes's sentences out in full, that is, including the actually operator and subscripting all the operators, even when this would be redundant, we see that a sentence of the form:

$$\ldots \Diamond_n(\ldots A_n Ra_1 \ldots a_m \ldots)\ldots$$

can be translated as:

$$\ldots \exists w_n(\ldots w_n Ra_1 \ldots a_m \ldots)\ldots$$

and a sentence of the form:

$$\ldots ARa_1 \ldots a_m \ldots$$

can be translated as:

$$\ldots w^* Ra_1 \ldots a_m \ldots$$

The worry is now apparent, for it seems that $\Diamond_n$ simply *is* $\exists w_n$, that A *is* w^*, and that A_n *is* w_n. At the very least, the burden of proof is on Forbes to show that his language is not simply a notational variant on its equivalent extensional language—in other words, that he isn't secretly quantifying over possible worlds.

Forbes's response is to maintain that his operators merely function as scope indicators. For example, in '$\Diamond_1\phi\Diamond_2 A_1\chi$', *syntactically* χ falls under the scope of the second diamond, yet the subscript shows that *semantically* it falls under the first. But this only pushes the worry one stage back. For quantifiers also indicate scope; and if Forbes is to maintain his insistence that his operators are not disguised quantifiers, he needs to give us an account of how else his operators accomplish this feat.

There are good reasons therefore to suggest that Forbes's translation does not constitute a genuine ontological reduction by paraphrase. Of course, this by itself does not refute Forbes's contention, for we have not yet shown *which* formulation has priority. It could be the case that it is the two-sorted extensional formulation that is an attempted ontological *inflation* by paraphrase of Forbes's formulation. However, Melia's second objection concerns how we come to understand the additional operators Forbes uses. These are initially introduced by way of a possible worlds model theory, which Forbes claims is simply a method for assigning truth values to modal sentences. This therefore implies that we have some independent grasp of the operators prior to the introduction of the model theory. But this seems incredible. Consider a statement of the form '$\Diamond_1\phi$'. This is understood as telling us to go to some possible world w, store it in the first position of an ordered sequence of possible worlds *w*, then see if ϕ is true at the possible world w. As Melia argues, there seems to be no way of understanding this without the model theory. Forbes's formulation therefore cannot be understood independently of the extensional formulation; consequently, it cannot have priority over the extensional formulation. It seems then that we should understand Forbes's formulation as the attempted paraphrase, and given the reasons to suppose such a paraphrase fails, it appears that Forbes's modal language actually invokes possible worlds.

Similarly then, any analogous strategy adopted by the constructive empiricist fictionalist would be vulnerable to similar objections. The subscripted operators would display the same syntactic structure as model theoretic quantifiers, and the introduction clauses for these operators would only be comprehensible if presented in a rich model theory. The purported extension of the modal language would then be parasitic on the notion of quantifying over models, and thus entirely unacceptable to the fictionalist. In fact, the problem is actually far more acute in the case of the mathematical fictionalist than it is for the primitive modalist. For what the Hazen/Melia argument shows in the case of modality is that an adequate modal language is committed to a possible worlds *model theory*. It would therefore be possible for the primitive modalist to concede the existence of abstract mathematical objects, while still denying the existence of possible worlds. Such a concession is clearly unavailable to the fictionalist however: for in the case of logical consistency, it is the existence of abstract mathematical objects that he denies.[8]

7. A deflationist justification of platonistic model theory

A final response I wish to consider for the constructive empiricist fictionalist is whether there is some way of *using* a model theoretic semantics, without endorsing any of the ontological commitments associated with it. Indeed, Field has proposed something alone these lines when he attempts to account for the applicability of mathematics in metalogical reasoning. It is worth pursuing then the possibility of extending this strategy to accommodate problems of expressive adequacy.

Field argues that a model theory is used to find out about logical possibility, via the following two schemata. The *model theoretic possibility* schema (MTP) states that:

If there is a model for A, then $\Diamond_L A$

[8] A more recent attempt to provide a deflationary translation of statements such as (1) and (2) is due to Nolan (2002). His proposal is to introduce an ontologically neutral quantifier, along the lines of Routley (1980), and hence to quantify over entities (possible worlds, models) that are held not to exist. However, as in the case of Forbes, it is far from clear how intelligible such a strategy is. Leaving aside the radical revision such a proposal requires for our understanding of the quantifiers, and the nominalistically dubious appeal to second-order quantification Nolan is forced to make, it must still be asked how these ontologically neutral quantifiers are to be understood. As with Forbes, it seems that the only way we are to understand the proposed paraphrase is in terms of traditional possible worlds semantics: a comparison Nolan himself invites, for example (2002), p. 55. Subsequently, it appears that Nolan's strategy is also parasitic on the notion of quantifying over worlds/models, and is thus similarly unacceptable to the fictionalist.

while the *model existence schema* (ME) states that:

If there is no model for A, then $\neg\Diamond_L A$

Clearly, as they stand, these schemata are useless for the fictionalist: since on this view there are no such things as models, MTP will always be trivially true, since its antecedent will always be false; and ME will be useless, since its antecedent will be true regardless of whether its consequent is true or false.

However, Field believes he can circumvent these difficulties by proposing acceptable deflationist surrogates. The idea is that a fictionalist can use a model theory provided it functions merely as a useful shorthand, in much the same way that he proposes to use mathematics in science. All that needs to be claimed is that *if* standard mathematics entails a model for A, then A is logically consistent. Since a conditional of this form makes no claim about whether standard mathematics is true, or whether such models do actually exist, accepting such claims carries no ontological commitments. Subsequently, the deflationist model theoretic possibility schema (MTP#) states that:

If $\Box_L$(mathematics is true $\supset$ there is a model for A), then $\Diamond_L A$

and similarly, the deflationist model existence schema (ME#) states that:

If $\Box_L$(mathematics is true $\supset$ there is no model for A), then $\neg\Diamond_L A$

Regardless of the success of such a strategy in accounting for the applicability of model theoretic reasoning in metalogic, it should be clear that this strategy is not going to solve the problem of expressive inadequacy as it stands. For one thing, the kind of claim that needs to be accounted for is not the right kind of expression for either MTP# or ME#. It is not that we need to posit the existence of a model to discover whether the claim is consistent; we need to posit the existence of models in order actually to *express* the claim. We might be able to utilise a model theory to show that $\Diamond_L A$ without being committed to the models, but we would still have the problem of not being able to formulate the content of A without subscripted modal operators, or model theoretic quantification.

A more promising proposal then would be to try to utilise some kind of deflationist schema to give the meaning of Forbes's subscripted operators in a way that does not depend on the existence of models. A sketch of such a schema, which attempts to give clauses for the subscripted operators in terms of the less contentious unsubscripted operators, would be of the form:

If $\Box_L$(mathematics is true $\supset$ there is a model m, a model m′ such that $\Diamond_L A$, and an ordered sequence of models *m* such that m′ is in the nth place of *m*), then $\Diamond_{Ln} A$

It is plausible that a series of schemata along these lines would succeed in giving clauses for the use of subscripted modal operators without invoking undue ontological commitment. But that is only half of the problem. We also need to know what these subscripted operators *mean*. Indeed, the only reason that the schemata introduced for the unsubscripted operators were acceptable was because there was an independent understanding of the operators, based on a primitive notion of implication. The schemata were justified because they were held to follow from the meaning of the unsubscripted operators. But a similar claim cannot be made in the case of the subscripted operators. We have no independent understanding of what they mean; consequently, the fictionalist cannot even justify his acceptance of such schemata, let alone invoke them to clarify his modal terminology.

In fact, whatever way the fictionalist looks at it, he is on to a loser. He cannot construct deflationist schemata to give the meaning of his operators since such schemata are only acceptable if the operators they introduce are already understood. But even if he *could* appeal to such schemata, he would undermine his position. The problem is that, for the fictionalist, model theoretic reasoning can be accounted for only so far as it serves as a shorthand. It is for finding out about logical possibility, not for giving it its content. But by the very fact that the model theory is being used to explain the modal operators, the fictionalist shows that in fact model theoretic reasoning does a lot more work than he supposes: it is in fact vital to the understanding of logical possibility, a *reductio* of his original claim. Either way, there appears to be no way of *understanding* the subscripted modal operators without commitment to models. And since the subscripted operators are necessary for a complete exposition of the central idea of logical possibility, fictionalism faces an irreducible commitment to the existence of mathematical objects.

8. The notion of consistency in scientific anti-realism

The problem with which I began this paper concerned what appeared to be the constructive empiricist's commitment to abstract mathematical objects, a commitment in direct conflict with his thoroughgoing empiricism. In response to this problem, I have examined at length Bueno's proposal that the constructive empiricist should adopt a (suitably modified) fictionalism towards these entities. What I hope the preceding has made clear is that such a response fails. What is of particular interest however is *why* such a response fails.

The crux of the issue turned on the notion of consistency: in order for the fictionalist programme to work, mathematics had to be shown to be conservative—a strong form of consistency. Unfortunately, we found that fictionalism does not have the resources to provide an adequate anti-realist account of this metalogical notion: there are some statements about consistency that we would wish to hold true, but which cannot be expressed within the fictionalist's primitive modal language. In fact, it appears that such statements can *only* be expressed by quantifying over abstract mathematical objects, and this raises an interesting challenge for any constructive empiricist response.

Essentially, the constructive empiricist faces the following dilemma. In order to give an adequate account of mathematical objects, he must also give an adequate account of various metalogical notions such as consistency. However, giving an adequate account of consistency either involves adopting a thoroughly realist semantics, and hence quantification over the very mathematical objects he is trying to do without; or it involves a substantial revision of our concept of consistency, by denying statements such as (2) for example.

I take it that revising our notion of consistency is not an option. The constructive empiricist is therefore left trying to reconcile a realist semantics about abstract mathematical objects with a deflationary ontology. But once the problem is set up in this way, an obvious solution presents itself. For recall the constructive empiricist's stance towards unobservable physical entities. The constructive empiricist is a semantic realist about his statements about unobservable physical entities: he takes these statements to be about the putatively referred to entities, and to be made true or false by the entities in question. However, the constructive empiricist also refuses to assert whether these entities really exist. Consequently he is not committed to holding these statements true, and thus is not committed to the existence of any unobservable entity that these statements quantify over. Instead, the constructive empiricist adopts a kind of 'committed agnosticism' towards his statements about unobservable physical entities, and this he argues is sufficient for using such statements in a variety of inferences. After all, committed agnosticism involves a substantial epistemological commitment, it 'involves a commitment to confront any future phenomena by means of the conceptual resources of [that which is accepted]...it is exhibited in the person's assumption of the role of explainer, in his willingness to answer questions *ex cathedra*' (van Fraassen, 1980, p. 12). That is to say, although the constructive empiricist does not believe the claims his theories make about unobservables, he is prepared to treat them *as if* they were true for the purposes of making predictions, explaining phenomena, and whatever else is required in his day-to-day scientific activity.

The crucial question then is how would a similar stance towards mathematical objects—realist semantics plus agnosticism about their existence—fare with respect to our original problem? Rosen's objection, recall, was vaguely transcendental: it argued that constructive empiricism entailed a form of nominalism, but that a necessary prerequisite for constructive empiricism was an explicit rejection of nominalism. The real test for the mathematical agnosticism response therefore concerns how well it applies at the 'transcendental' level; or in other words, in how intelligible it is for the constructive empiricist to be agnostic about mathematical objects *while in the process of stating what constructive empiricism amounts to*.

In his original paper, Rosen (1994, pp. 167–168) considers just such a response. His conclusion however is that this sort of strategy cannot work. He argues that if the constructive empiricist is going to be agnostic about mathematical objects, then 'to accept [a theory] is to believe that the world is such that *if there were such a thing as* [the theory], *it would be empirically adequate*' (ibid., p. 167; original emphasis). It follows then that if the constructive empiricist is going to adopt the mathematical agnosticism strategy, then in order to state what constructive empiricism amounts to, he must assert a counterfactual. Which is just to say that the immediate consequence of mathematical agnosticism is that the constructive empiricist's statement of his position must be essentially *modal* in nature.

But this, according to Rosen, raises a serious problem. He argues that:

> The trouble with all of this from van Fraassen's perspective is modality itself...there is reason to think that the constructive empiricist is committed to agnosticism about a range of modal facts. The counterfactuals that constitute the objects of belief on the present proposal arguably fall into that class. And if that is right then whatever its merits, the proposal is not available to van Fraassen. (Rosen, 1994, p. 168)

So according to Rosen, agnosticism about mathematical objects commits the constructive empiricist to a modal statement of his position; but since the constructive empiricist must also be agnostic about this modal statement, it is a statement he cannot believe. Mathematical agnosticism therefore entails the unfortunate consequence that the constructive empiricist can no longer believe what his position is supposed to be.

Fortunately however, Rosen's argument can be resisted. For what Rosen underestimates is the degree of *epistemological commitment* associated with the constructive empiricist's agnosticism. For consider again the case of unobservables. The constructive empiricist refuses to assert whether such entities exist, and thus whether statements about such entities are true or false. But as noted above, he maintains that by adopting an attitude of committed agnosticism towards such statements, he earns the right to use them in a variety of inferences. Similarly then, by adopting an attitude of committed agnosticism towards his modal statements, the constructive empiricist can also earn the right to use the sort of counterfactuals Rosen believes him to be committed to, and without any undue difficulty.[9]

It seems then that mathematical agnosticism *is* a proposal available to the constructive empiricist. And moreover, it seems that adopting an attitude of committed agnosticism towards abstract mathematical objects is the most attractive response available to Rosen's objection. For in so doing, it appears that the constructive empiricist can solve all of his problems. By remaining agnostic about the existence of mathematical objects, he can avoid any problematic ontological commitment to such entities, in keeping with his general nominalism. Also, given the substantial epistemological commitments associated with such an agnosticism, he earns the right to continue to *use* statements about mathematical objects, and thus avoids any 'transcendental' difficulties. And most importantly, since his committed agnosticism is couched in a thoroughly realist semantics, he can continue to take a quantificational approach to metalogical notions such as consistency.

Of course, the success of such a strategy does depend heavily on whether the constructive empiricist's purported distinction between belief and committed agnosticism can be maintained; and this has been questioned in the literature (e.g. Horwich, 1991). However, it is worth considering what is at stake here. The agnosticism response sketched here yields an anti-realist account of mathematical objects, provided that a distinction can be drawn between two allegedly different doxastic attitudes. This may not be easy, but it appears to be a much more promising goal than that required by Bueno's proposal: fictionalism, by contrast, yields an anti-realist account of mathematical objects, provided that we can reinterpret the applicability of mathematical statements in terms of conservativeness, sketch an alternative (primitively modal) semantics for our various metalogical concepts, and in the process defend a substantial revision of our notion of consistency. Whatever the prospects for the agnostic's epistemology, it strikes me as considerably more plausible than the fictionalist's metalogic.

There is a general point to be made here; and again, a comparison with debates over the metaphysics of modality will help to explicate it. In a recent paper, Divers (2004) has argued that agnosticism about possible worlds promises substantial advantages over other anti-realist positions about modality. This is because the principle alternatives—the primitive modalism of Forbes that we have already encountered, as well as Rosen's

[9] The issue of the constructive empiricist's attitude towards modal statements is thoroughly discussed in the exchange between Ladyman (2000, 2004) and Monton & van Fraassen (2003). Essentially, Ladyman presents the constructive empiricist with the following dilemma: either his modal statements are made true by objective modal facts, in which case his epistemological principles prevent him from believing them; or his modal statements are not made true by objective modal facts, in which case they are too arbitrary to support the distinction between observable and unobservable states of affairs. The constructive empiricist can resolve this dilemma however by adopting a form of modal agnosticism—analogous to his attitude towards unobservable physical entities—as sketched above. In simple terms, since the constructive empiricist need only be *committed* to his modal statements in order to *use* them, the fact that he cannot *believe* such statements is irrelevant, and Ladyman's dilemma is dissolved.

(1990) fictionalism, and Blackburn's (1986) expressivism—all involve a reinterpretation of the semantics of our modal discourse. The major challenge for these strategies then is to demonstrate that the various reinterpretations proposed manage to preserve the inferential roles played by our modal discourse when interpreted at face value. And this is by no means an easy task; as we saw with the case of Forbes's modalism (Section 6), there are certain intuitively true modal statements that cannot be expressed in his primitive modal language without invoking the various entities his account is meant to reject.

Given then the close analogy between possible world semantics and metalogical model theory that has driven much of this paper, we can suggest a similar conclusion about the notion of consistency. Divers argues that modal agnosticism offers a substantial advantage over other modal anti-realisms, on the grounds that it avoids a difficult (and unpromising) reinterpretation of the semantics of our modal discourse. Similarly, I believe that a constructive empiricist agnosticism about mathematical objects within the philosophy of science offers similar advantages, for it does not require a reinterpretation of our various metalogical discourses. This has been shown in the case of (partial structures) fictionalism; my claim is that it holds for other anti-realist proposals too.

9. Conclusion

The constructive empiricist faces the problem that he seems committed to believing in the existence of abstract objects, in direct violation of his explicit nominalism. In response, Bueno has argued that the constructive empiricist can avoid these ontological commitments by adopting a (suitably modified) fictionalism about mathematics. The main aim of this paper has been to show that such a strategy is untenable.

Instead, I believe that the correct response for the constructive empiricist is to adopt an attitude of committed agnosticism towards abstract mathematical objects, in the same way as he is agnostic about unobservable physical objects. Moreover, I think that such a response has considerable advantages over other anti-realist proposals, on the grounds that it replaces the rather unpromising challenge of offering a radical reinterpretation of our metalogical discourse with the far more tractable task of outlining and defending the epistemological distinction between agnosticism and belief.

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